

EXERCISES

For more practice, see *Extra Practice*.

Practice and Problem Solving

A Practice by Example

Example 1 (page 4)

Find a pattern for each sequence. Use the pattern to show the next two terms.

1. 5, 10, 20, 40, ...
2. 3, 33, 333, 3333, ...
3. 1, -1, 2, -2, 3, ...
4. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$
5. 15, 12, 9, 6, ...
6. 81, 27, 9, 3, ...
7. O, T, T, F, F, S, S, E, ...
8. J, F, M, A, M, ...
9. 1, 2, 6, 24, 120, ...
10. 2, 4, 8, 16, 32, ...
11. $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \dots$
12. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$
13. George, John, Thomas, James, ...
14. Martha, Abigail, Martha, Dolley, ...
15. George, Thomas, Abe, Alexander, ...
16. Aquarius, Pisces, Aries, Taurus, ...

Draw the next figure in each sequence.



Example 2 (page 5)

Use the table and inductive reasoning. Make a conjecture about each value.

19. the sum of the first 6 positive even numbers
20. the sum of the first 30 positive even numbers
21. the sum of the first 100 positive even numbers

2	= 2 = 1 · 2
2 + 4	= 6 = 2 · 3
2 + 4 + 6	= 12 = 3 · 4
2 + 4 + 6 + 8	= 20 = 4 · 5
2 + 4 + 6 + 8 + 10	= 30 = 5 · 6

22. Use the pattern in Example 2 to make a conjecture about the sum of the first 100 odd numbers.

Predict the next term in each sequence. Use your calculator to verify your answer.

23. $12345679 \times 9 = 111111111$
 $12345679 \times 18 = 222222222$
 $12345679 \times 27 = 333333333$
 $12345679 \times 36 = 444444444$
 $12345679 \times 45 = \blacksquare$
24. $1 \times 1 = 1$
 $11 \times 11 = 121$
 $111 \times 111 = 12321$
 $1111 \times 1111 = 1234321$
 $11111 \times 11111 = \blacksquare$

Example 3 (page 5)

Find one counterexample to show that each conjecture is false.

25. The sum of two numbers is greater than either number.
26. The product of two positive numbers is greater than either number.
27. The difference of two integers is less than either integer.
28. The quotient of two proper fractions is a proper fraction.

Example 4 (page 6)

29. **Weather** The speed with which a cricket chirps is affected by the temperature. If you hear 20 cricket chirps in 14 seconds, what is the temperature?

Chirps per 14 Seconds

5 chirps	45°F
10 chirps	55°F
15 chirps	65°F

- 30. Physical Fitness** Dino works out regularly. When he first started exercising, he could do 10 push-ups. After the first month he could do 14 push-ups. After the second month he could do 19, and after the third month he could do 25. Predict the number of push-ups Dino will be able to do after the fifth month of working out. How confident are you of your prediction? Explain.

B Apply Your Skills

Find a pattern for each sequence. Use the pattern to show the next two terms.

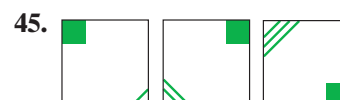
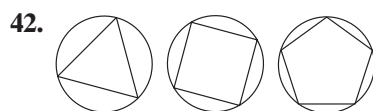
31. 1, 3, 7, 13, 21, ... 32. 1, 2, 5, 6, 9, ... 33. 0.1, 0.01, 0.001, ...
 34. 2, 6, 7, 21, 22, 66, 67, ... 35. 1, 3, 7, 15, 31, ... 36. $0, \frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \dots$
 37. M, V, E, M, ... 38. AL, AK, AZ, AR, ... 39. H, He, Li, Be, ...



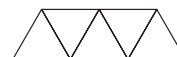
- 40. Writing** Choose two of the sequences in Exercises 31–36 and describe the patterns.

- 41.** Draw two parallel lines on your paper. Locate four points on the paper, each an equal distance from both lines. Describe the figure you get if you continue to locate points, each an equal distance from both lines.

Draw the next figure in each sequence.



- 46.** Use inductive reasoning. Find the perimeter when 100 triangles are put together in the pattern shown. Assume that all triangle sides are 1 cm long.



- 47. Math in the Media** Read this excerpt from a news article.


Top female runners have been improving about twice as quickly as the fastest men, a new study says. If this pattern continues, women may soon outrun men in competition!


The study is based on world records collected at 10-year intervals, starting in 1905 for men and in the 1920s for women. If the

trend continues, the top female and male runners in races ranging from 200 m to 1500 m might attain the same speeds sometime between 2015 and 2055.

Women's marathon records date from 1955 but their rapid fall suggests that the women's record will equal that of men even more quickly, perhaps by 2005.


- What conclusion was reached in the study?
- How was inductive reasoning used to reach the conclusion?
- Explain why the conclusion that women may soon be outrunning men may be incorrect. For which race is the conclusion most suspect? For what reason?

-  **48. Communications** The table shows the number of commercial radio stations in the United States for a 50-year period.
- Make a line graph of the data.
 - Use the graph and inductive reasoning to make a conjecture about the number of radio stations in the United States in the year 2010.
 - How confident are you about your conjecture? Explain.
- 49. Open-Ended** Write two different number-pattern sequences that begin with the same two numbers.



Number of Radio Stations


1950	2,835
1960	4,224
1970	6,519
1980	7,871
1990	9,379
2000	10,577






SOURCE: Federal Communications Commission


- 50. Error Analysis** For each of the past four years, Paulo has grown 2 in. every year. He is now 16 years old and is 5 ft 10 in. tall. He figures that when he is 22 years old he will be 6 ft 10 in. tall. What would you tell Paulo about his conjecture?

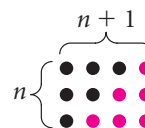
- 51. Coordinate Geometry** You are given x - and y -coordinates for 14 points.
- $A(1, 5)$ $B(2, 2)$ $C(2, 8)$ $D(3, 1)$ $E(3, 9)$ $F(6, 0)$ $G(6, 10)$
 $H(7, -1)$ $I(7, 11)$ $J(9, 1)$ $K(9, 9)$ $L(10, 2)$ $M(10, 8)$ $N(11, 5)$
- Graph each point.
 - Most of the points fit a pattern. Which points do not?
 - Describe the figure that fits the pattern.

-  **52. History** Leonardo of Pisa (about 1175–1258), also known as Fibonacci (fee buh NAH chee), was born in Italy and educated in North Africa. He was one of the first Europeans known to use modern numerals instead of Roman numerals. The special sequence 1, 1, 2, 3, 5, 8, 13, ... is known as the Fibonacci sequence. Find the next three terms of this sequence.

-  **53. Time Measurement** Leap years have 366 days.
- The years 1984, 1988, 1992, 1996, and 2000 are consecutive leap years. Look for a pattern in their dates. Then, make a conjecture about leap years.
 - Of the years 2010, 2020, 2100, and 2400, which do you think will be leap years?
 - Research** Find out whether your conjecture for part (a) and your answer for part (b) are correct. How are leap years determined?

-  **Challenge**  **54. History** When he was in the third grade, German mathematician Karl Gauss (1777–1855) took ten seconds to sum the integers from 1 to 100. Now it's your turn. Find a fast way to sum the integers from 1 to 100; from 1 to n . (*Hint:* Use patterns.)

-  **55. a. Algebra** Write the first six terms of the sequence that starts with 1, and for which the difference between consecutive terms is first 2, and then 3, 4, 5, and 6.
- b.** Evaluate $\frac{n^2 + n}{2}$ for $n = 1, 2, 3, 4, 5$, and 6. Compare the sequence you get with your answer for part (a).
- c.** Examine the diagram at the right and explain how it illustrates a value of $\frac{n^2 + n}{2}$.
- d.** Draw a similar diagram to represent $\frac{n^2 + n}{2}$ for $n = 5$.



Need Help?

For Exercise 51, you may want to review "Coordinates of a point" in the Glossary.



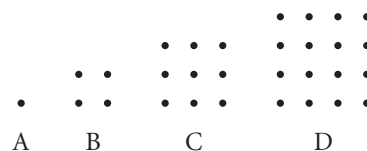
Standardized Test Prep

Multiple Choice

56. The sum of the numbers from 1 to 10 is 55. The sum of the numbers from 11 to 20 is 155. The sum of the numbers from 21 to 30 is 255. Based on this pattern, what is the sum of numbers from 91 to 100?
- A. 855 B. 955 C. 1055 D. 1155
57. Which of the following conjectures is false?
- F. The product of two even numbers is even.
 G. The sum of two even numbers is even.
 H. The product of two odd numbers is odd.
 I. The sum of two odd numbers is odd.

Short Response

58. a. How many dots would be in each of the next three figures?
 b. Write an expression for the number of dots in the n th figure.



Extended Response



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59. a. Describe the pattern. List the next two equations in the pattern.
 b. Guess what the product of 181 and 11 is. Test your conjecture.
 c. State whether the pattern can continue forever. Explain.

$$\begin{aligned} (101)(11) &= 1111 \\ (111)(11) &= 1221 \\ (121)(11) &= 1331 \\ (131)(11) &= 1441 \\ (141)(11) &= 1551 \end{aligned}$$

Mixed Review

Previous Course

Graph each point.

60. $Y(-5, -8)$ 61. $B(7, -10)$ 62. $M(9, 12)$ 63. $Q(-3, 2)$
 64. $G(-6, 0)$ 65. $F(-4, -5)$ 66. $C(-7, 10)$ 67. $N(0, -5)$

Classify the points in Exercises 60–67 as described below.

68. in Quadrant IV 69. on the y -axis 70. on the x -axis